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The impact of power transformations on the parameters of the gamma distributed error component of a multiplicative error model

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Abstract. Considering that the error component of a Multiplicative Error Model(MEM) can possibly be a gamma distribution ($G(\alpha, \beta)$; α and β are shape and scale parameters respectively). This paper studies the effects of power transformations on the mean and variance of a gamma distributed error component. The popular transformations: square-root, Wilson-Hilferty, inverse-square-root, inverse, inverse-square and square transformations were studied. The probability density function(pdf) and the kth-uncorrected moment of the p-th power –transformed gamma random variable are obtained. The mean and variance of $G(\alpha, \beta)$ and those of the power – transformed distributions are calculated for $\alpha = 5, 6, \cdots, 99, 100$ with the corresponding values of β for which the mean of the untransformed distribution is equal to one. The effects of the power transformations on the mean and variance of the gamma distribution are investigated for $\alpha \geq 9$ where all the transformed distributions have a unit mean value. The relative changes in mean and variance are used for the investigations. For all the transformations, there are no changes in the mean. For variances: it was found that there are relative increases for the inverse, inverse square and square transformations. However, the square-root, Wilson Hilferty and inverse-square-root transformations decreased the variance relative to the variance of the untransformed distribution. This paper concludes that the square-root, Wilson Hilferty and inverse-square-root transformations would yield better results when using MEM that the error component assumes a gamma distribution and where the goal is to stabilize the variance of the data set through data transformation.

Keywords: multiplicative error model, gamma distribution, power transformation, mean, variance.

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1. Introduction

The Multiplicative Error Models (MEM), which are particularly suited to model non-negative time series, were first introduced as Autoregressive Conditional Duration (ACD) models by Engle and Russell (1998) and generalized to any non–negative valued process by Engle (2002).

Suppose $\{x_t\}$ is a discrete time process defined on $[0, +\infty), t \in N$, where N is the set of Natural numbers and let Ψ_{t-1} be the information available for forecasting x_t. According to Brownless et al. (2011), $\{x_t\}$ follows a MEM if it can be represented as

$$x_t = \mu_t \epsilon_t \tag{1}$$

where, conditionally on Ψ_{t-1}: μ_t is a positive quantity that evolves deterministically according to a parameter vector θ,

$$\mu_t = \mu(\theta, \Psi_{t-1}) \tag{2}$$

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ϵ_t is a random variable (rv) with probability density function (pdf) defined over a $[0, +\infty)$ support, unit mean and unknown constant variance,

$$\epsilon_t|\Psi_{t-1} \sim D^+(1, \sigma^2) \tag{3}$$

where D^+ is any probability distribution defined over a $[0, \infty)$ support. There is no question that the conditional distribution of ϵ_t in (1.1) can be specified by any distribution with the characteristics in (1.3) such as Gamma, Log-Normal, Weibull, Inverted-Gamma and mixtures of them (Bauwens and Giot (2000), Engle and Gallo (2006), Lanne (2006) and De Luca and Gallo (2010)). It is important to mention that the left-truncated normal distribution has the property in (1.3) of which the effects of logarithm , inverse, square, inversesquare-root and square-root transformations on the error component of traditional multiplicative time series model have been studied (Iwueze(2007); Nwosu et al. (2013); Ohakwe et al.(2013); Ajibade et al. (2015) and Ohakwe et al. (2018)).

In statistical modeling of a data set using MEM, the basic assumptions of ϵ_t is unit mean and constant variance, σ^2. This is not always true in reality and as a result some remedial measures have to be made to ensure conformance to the required assumptions. One of the remedial measures that has attracted so much attention by researchers is data transformation (Thoeni (1967) and Hoyle (1973)). Data transformation is the replacement of a variable by a function of that variable that can change the shape of a distribution or relationship.

Power transformation is a class of data transformations. The popular forms of the transformations are: square-root, Wilson-Hilferty, inverse-square-root, inverse, inverse-square and square transformations. Let the error term ϵ_t in (1.1) be a Gamma (Ga) distribution with unit mean and constant variance (that is, $\epsilon_t \sim Ga(1, \sigma^2)$), Ohakwe et al. (2012) and Ohakwe (2013) studied the effects of square root and inverse transformations on the means and variances of a gamma-distributed error component of a MEM respectively. In Ohakwe et al. (2012), it was found that the mean is one and that the ratio of the variance of the untransformed gamma distributed error component with unit mean to that of the square-root transformed is approximately 4. Ohakwe (2013) found decreases in the unit mean and variance and went further to model the changes in the mean and variance between the untransformed and the inverse-transformed distributions for various values of the shape parameter $\alpha = 2, 3, \cdots, 12$. A cubic relationship was obtained for the change in mean, while a quadratic relationship was obtained for the change in variance. Ohakwe et al (2013) used the probability density function (pdf) of inverse transformed gamma distribution given in Cook(2008) whose pdf denoted as $f(\epsilon_t)$ was obtained from the gamma distribution with the following parameterization

$$f(\epsilon_t) = \frac{\beta^\alpha (\epsilon_t)^{\alpha-1} e^{-\beta\epsilon_t}}{\Gamma(\alpha)}, \epsilon_t > 0, \Gamma(\alpha) > 0, \tag{4}$$

where ϵ_t is the error component of the MEM in (1.1). The gamma distribution can also assume the following parameterization (Bhaumik et al. (2009)).

$$f(\epsilon_t) = \frac{(\epsilon_t)^{\alpha-1} e^{-\frac{\epsilon_t}{\beta}}}{\Gamma(\alpha)\beta^\alpha}, \epsilon_t > 0, \Gamma(\alpha) > 0, \beta^\alpha > 0 \tag{5}$$

where α and β are the shape and scale parameters respectively.

The approach adopted by Ohakwe et al.(2012) was simply obtaining the variance ratio of the untransformed and the transformed distributions while Ohakwe (2013) used the inverse transformed Gamma distribution obtained from (1.4). This study aims to generalize the p-th power transformation using (1.5) with specific interest on the most commonly used power transformations in the literature (Wilson an Hilferty (1931); Tukey(1977); Nwosu et al (2013);. Ohakwe et al (2013); Ibeh and Nwosu (2013); Ajibade et al (2015); Dike et al. (2015);Ohakwe et al. (2018)). The effect of the commonly used transformations on the unit mean and variance of a gamma-distributed error component of a MEM is studied. The remaining part of this paper is organized as follows: Section Two contains the the p-th power transformed gamma distributed variable together with its k-th uncorrected moment from where the mean and variance functions for the study transformations would be obtained. Computations of the means, variances and relative changes in means and variances of the transformed distributions for various values of the shape and scale parameters are contained in Section Three. Discussions of the results are contained in Section Four while the conclusion is contained in Sections Five.

2. P-th transformed gamma distributed random variable

Power transformation is a form of transformation that is frequently used in statistical analysis(Ozdemir et al (2017). Furthermore, Ozdemir et al (2017) defined a power transformation as follows;

$$Y = \begin{cases} X^p, & p \neq 0 \\ \log(X), & p = 0 \end{cases} \tag{6}$$

where Y is the transformed variable, X, the untransformed variable and p, a constant. In this study we shall consider when $p \neq 0$. Our choice of p-th power transformation is based on the fact that the commonly used power transformations in the literature are subclasses of it. Suppose $Y_t = \epsilon_t^p$,

$$\epsilon_t = Y_t^{\frac{1}{p}} \tag{7}$$

and

$$|J| = \left|\frac{d\epsilon_t}{dy_t}\right| = \left|\frac{1}{p} y_t^{\frac{1}{p}-1}\right| = \left|\frac{1}{p}\right| y_t^{\frac{1}{p}-1} \tag{8}$$

where $|J|$ is the absolute value of the Jacobian of the p-th power transformation. The pdf of Yt, denoted as $f(y_t)$ is then obtained from (1.5) as $f(y_t) = f\left(\epsilon_t = y_t^{\frac{1}{p}}\right)\left|\frac{d\epsilon_t}{dy_t}\right|$ (Ramachandran and Tsokos (2009), thus

$$f(y_t) = \frac{y_t^{\frac{\alpha}{p}-1} e^{-\frac{y^{\frac{1}{p}}}{\beta}}}{|p|\Gamma(\alpha)\beta^{\alpha}}, y_t > 0 \tag{9}$$

Equation (2.4) can be shown to be a proper pdf by simply establishing that

$$\int_0^{\infty} f(y_t) dy_t = 1 \tag{10}$$

The popular transformations for various values of p is given in Table 1.

Table 1.: The popular transformations for various values of p

SN	P	Type of Transformation
1	1	No Transformation
2	$\frac{1}{2}$	Square - root Transformation
3	$\frac{1}{3}$	Wilson- Hilferty Transformation
4	$-\frac{1}{2}$	Inverse –square - root Transformation
5	-1	Inverse Transformation
6	-2	Inverse – Square Transformation
7	2	Square Transformation

Given (2.4), the k-th uncorrected moment is obtained as

$$E(Y_t^k) = \int_0^\infty y_t^k f(y_t)dy_t = \frac{\beta^{pk}\Gamma(\alpha+pk)}{\Gamma(\alpha} \tag{11}$$

The summary of results for k =1, 2 are given in Table 2

Table 2.: The The Expectation of Y_t^k for k = 1, 2

K	$E(Y_t^k)$	Functional Expression
1	$E(Y_t) = \mu_{y_t}$	$\frac{\beta^p\Gamma(\alpha+p)}{\Gamma(\alpha)}$
2	$E(Y_t^2)$	$\frac{\beta^{2p}\Gamma(\alpha+2p)}{\Gamma(\alpha}$

The means and variances for the various power-transformed distributions are obtained by putting the values of p into the expressions given in Table 2, where mean

$$(\mu_{y_t}) = E(Y_t)$$

and Variance

$$(\sigma_{y_t}^2) = E(Y_t^2) - [E(Y_t]^2$$

and the results are given in Table 3. It is important to mention that all computations involving α

Table 3.: The popular transformations for various values of p and their Means and Variances

SN	P	Transformed Distribution	Mean (E(Y$_t$)) =μ_{yt}	Variance (Var(Y$_t$)) = σ_{yt}^2
1	$\frac{1}{2}$	Square - root (SR)	$\frac{\beta^{\frac{1}{2}}\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha)}$	$\alpha\beta - \left[\frac{\beta^{\frac{1}{2}}\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha)})\right]^2$
2	$\frac{1}{3}$	Wilson- Hilferty(WH)	$\frac{\beta^{\frac{1}{3}}\Gamma(\alpha+\frac{1}{3})}{\Gamma(\alpha)}$	$\frac{\beta^{\frac{2}{3}}\Gamma(\alpha+\frac{2}{3})}{\Gamma(\alpha)} - \left[\frac{\beta^{\frac{1}{3}}\Gamma(\alpha+\frac{1}{3})}{\Gamma(\alpha)}\right]^2$
3	$-\frac{1}{2}$	Inverse –square - root (ISR)	$\frac{\beta^{-\frac{1}{2}}\Gamma(\alpha-\frac{1}{2})}{\Gamma(\alpha)}$	$\frac{\beta^{-1}}{\alpha-1} - \left[\frac{\beta^{-\frac{1}{2}}\Gamma(\alpha-\frac{1}{2})}{\Gamma(\alpha)}\right]^2$
4	-1	Inverse (I)	$\frac{\beta^{-1}}{\alpha-1}$	$\frac{\beta^{-2}\Gamma(\alpha-2)}{\Gamma(\alpha)} - \left[\frac{\beta^{-1}}{\alpha-1}\right]^2, \alpha > 2$
5	-2	Inverse - Square (IS)	$\frac{\beta^{-2}\Gamma(\alpha-2)}{\Gamma(\alpha)}$	$\frac{\beta^{-4}\Gamma(\alpha-4)}{\Gamma(\alpha)} - \left[\frac{\beta^{-2}\Gamma(\alpha-2)}{\Gamma(\alpha)}\right]^2, \alpha > 4$
6	2	Square (S)	$\frac{\beta^{2}\Gamma(\alpha+2)}{\Gamma(\alpha)}$	$\frac{\beta^{4}\Gamma(\alpha+2)}{\Gamma(\alpha)} - \left[\frac{\beta^{2}\Gamma(\alpha+2)}{\Gamma(\alpha)}\right]^2$

will be greater than 4 as a result of the fact that the variance of the inverse – square transformed distribution is undefined for $\alpha = 4$ (see number 5 of Table 3).

3. Relative change in means and variances of the two distributions

In this Section, we would first obtain the means and variances of the transformed and the untransformed distributions that would be used to compute the relative changes in means and variances between the untransformed and transformed distributions.

The mean (μ_{ϵ_t}) and variance ($\sigma^2_{\epsilon_t}$) of (1.5) (that is the untransformed distribution) are respectively obtained as

$$\mu_{\epsilon_t} = \int_0^\infty \epsilon_t f(\epsilon_t) d\epsilon_t = \alpha\beta \tag{12}$$

and

$$\sigma^2_{\epsilon_t} = \left[\int_0^\infty \epsilon_t^2 f(\epsilon_t) d\epsilon_t \right] - [\mu_{\epsilon_t}]^2 = \alpha\beta^2 \tag{13}$$

Considering the unit mean assumption required for modeling, we simply calculate the theoretical values of μ_{ϵ_t} ,$\sigma^2_{\epsilon_t}$, μ_{y_t} and $\sigma^2_{y_t}$ using values of $\alpha = 5, 6, \cdots, 100$ and corresponding values of β for which $\mu_{\epsilon_t} = 1.0$. From (3.1), for $\mu_{\epsilon_t} = 1.0$, without loss of generality, we can either adopt $\alpha = \frac{1}{\beta}$ or $\beta = \frac{1}{\alpha}$. However, in order to maintain the values of the shape parameters as positive integers, we will adopt $\beta = \frac{1}{\alpha}, \forall\alpha$ for all the computations involving the untransformed and transformed distributions and the results are given in Tables 4 and 5 (see appendix) . Table 4 contains the mean of the transformed and the untransformed distributions while their variances are contained in Table 5.

Considering that the interest in this study is to examine the effect of the various power transformations on the Gamma distributed error term as regards the mean and the variance, we will simply use the relative change in means and variances between the untransformed and the transformed distributions in measuring the effect of a transformation.

An indicator of the relative difference between two variables X and Y (or the change from X to Y) is defined as a real-valued function $C(x, y)$, defined for all positive X and Y, $C : R^2_+ \to R$, which has the following properties (Vartia (1976, p. 9-25));

(1) $C(x, y) = 0, iff\ x = y$
(2) $C(x, y) > 0, iff\ y > 0$
(3) $C(x, y) < 0, iff\ y < x$
(4) $\forall a : a > 0 \Rightarrow C(ax, ay) = C(x, y)$

Between two variables X and Y, Tornqvist, Vartia and Vartia (1985) described the following;

$$\frac{Y - X}{X} \tag{14}$$

as the customary indicator of relative change among other indicators and this we would adopt in this study. For the effect on the mean, the two variables of interest are μ_{ϵ_t} and μ_{y_t} and by adopting the measure in (3.3) we would calculate the Relative Change in mean (RCIM) using

$$RCIM = \frac{\mu_{y_t} - \mu_{\epsilon_t}}{\mu_{\epsilon_i}} = \mu_{y_t} - 1.0 \tag{15}$$

where RCIM ¿ 0 indicates increase, RCIM = 0 indicates no change and RCIM ¡ 0 indicates decrease in mean.
Furthermore, for the effect on the variance, the determinant variables are $\sigma^2_{\epsilon_t}$ and $\sigma^2_{y_t}$ and by also adopting (3.3) we would calculate the Relative Change in Variance (RCIV) between the transformed and the untransformed distributions using

$$RCIV = \frac{\sigma^2_{y_t} - \sigma^2_{\epsilon_t}}{\sigma_{\epsilon_t^2}} \tag{16}$$

Here $RCIV > 0$ indicates increase, RCIV = 0 indicates no change and RCIV < 0 indicates decrease in variance. Considering that the theoretical means of the transformed distributions are approximately 1.0 to the nearest whole number for $\alpha \geq 9$ as shown in Table 4, we therefore compute the RCIM and RCIV values for $\alpha \geq 9$. Furthermore, the plots of the variances of the untransformed and transformed distributions against the shape parameter values, $\alpha \geq 9$ are given in Figure 1 (see appendix) while the computations of the RCIV are contained in Table 6 (see appendix). Finally, the plots of the RCIV against α are given in Figure 2 (see appendix).

4. Results and discussions

The results in Table 4 indicate that the unit-mean assumption is unaffected by the transformations. This result is in agreement with the findings of Ohakwe et al., (2012).

For the variances given in Table 5, the variances for the inverse(VarIS), square(VarS) and inverse (VarI) of the transformed distributions are higher than the variance of the untransformed distribution (VarUN) while those of the square root(VarSR), inverse square root(VarISR) and Wilson Hilferty (VarWH) are lower than that of the untransformed distribution. These higher and lower variances can be clearly seen in Figure 1, where the variances for the inverse square, square and inverse transformations are above the variance of the untransformed distribution while those of the square root, inverse square root and Wilson Hilferty are below that of the untransformed distribution.

In Figure 2, the RCIV for ISR, WH and SR transformations are all less than zero which indicate reduced variances under such transformations while those for I, IS and S transformations are all greater than zero which indicate increased variance resulting from such transformations. However, WH transformation has the highest magnitude of decrease in variance with factors ranging from -8.0061 to (-8.0000), followed by SR transformation with range, -3.0579 to (-3.0050) and then inverse-square-root transformation with range, -2.9254 to (-2.2122).

Furthermore, for the inverse (I), inverse – square (IS) and square (S) transformations that yielded higher variances, the RCIV have ranges: 0.7752 – 0.9387 for IS, 0.7561 – 0.8071 for S and 0.0395 – 0.3855 for I. Finally, it is important to mention that stability of the variances for all the transformations is achieved from the point, $\alpha \geq 17$, where the variances for all the transformations are all approximately zero.

5. Conclusion

In this study we have investigated the effect of the commonly used power transformations namely: square-root, Wilson-Hilferty, inverse-square-root, inverse, inverse-square and square transformations on the unit-mean and variance of the Error component of a multiplicative Error model(MEM) which has a gamma distribution that requires a variance-stabilization transformation. The probability density function(pdf) and the kth-uncorrelated moment of the p-th power –transformed gamma random variable were derived. Also the functional expressions for the mean and variance of the Gamma distribution under the commonly used power transformations were established. The means and variances of the gamma distribution $(G(\alpha, \beta))$ and those of the p-th power – transformed distributions were calculated for $\alpha = 5, 6, \cdots, 99, 100$ with the corresponding values of β for which the mean of the untransformed distribution is equal to one. For uniformity of results the impact of the various transformations were investigated with respect to $\alpha = 9, 10, \cdots, 99, 100$, where the means of all the transformed distributions are approximately equal to 1.0 to the nearest whole number. For all the transformations, there was no relative change in the mean between the transformed and the untransformed distributions. For the effects on variances: it was found that there are relative increases in variances within the ranges; 0.7561 – 0.8071, 0.7752 – 0.9387 and 0.0395 – 0.3855 for the inverse, the inverse-square and the square transformations respectively. For the square-root, the Wilson Hilferty and the inverse-square-root transformations there were decreases in variances relative to the variance of the untransformed distribution. The relative decreases in variances are within the ranges: -3.0579 – (-3.0050), -8.0061 – (-8.0000) and -2.9254 – (-2.2122) for the square-root, the Wilson Hilferty and the inverse-square-root transformations respectively. In summary, it was established in this study that there are no effects on the mean as a result of the various transformations. However the inverse, the inverse-square and the square transformations increased the variance while the square-root, the Wilson Hilferty and the inverse-square-root transformations decreased the variance. We conclude that the square-root, Wilson Hilferty and inverse-square-root transformations that decreased the variance (RCIV < 0) will yield better results when using MEM whose error component assumes a gamma distribution and where data transformation is required to stabilize the variance of the data set through data transformation.

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Appendix

APPENDIX

Table 4 : Means of the untransformed and Transformed Distributions

α	$\beta=1/\alpha$	μ_{ε_i}	Mean of the transformed Distribution $((\mu_{y_i})$					
			SR	WH	ISR	I	IS	S
5	0.2000	1.0	0.9754	0.9778	1.0837	1.2500	2.0833	1.2000
6	0.1667	1.0	0.9794	0.9815	1.0684	1.2000	1.8000	1.1667
7	0.1429	1.0	0.9823	0.9841	1.0579	1.1667	1.6333	1.1429
8	0.1250	1.0	0.9845	0.9861	1.0501	1.1429	1.5238	1.1250
9	0.1111	1.0	0.9862	0.9877	1.0442	1.1250	1.4464	1.1111
10	0.1000	1.0	0.9876	0.9889	1.0396	1.1111	1.3889	1.1000
11	0.0909	1.0	0.9887	0.9899	1.0358	1.1000	1.3444	1.0909
12	0.0833	1.0	0.9896	0.9907	1.0327	1.0909	1.3091	1.0833
13	0.0769	1.0	0.9904	0.9915	1.0301	1.0833	1.2803	1.0769
14	0.0714	1.0	0.9911	0.9921	1.0278	1.0769	1.2564	1.0714
15	0.0667	1.0	0.9917	0.9926	1.0259	1.0714	1.2363	1.0667
16	0.0625	1.0	0.9922	0.9931	1.0242	1.0667	1.2190	1.0625
17	0.0588	1.0	0.9927	0.9935	1.0228	1.0625	1.2042	1.0588
18	0.0556	1.0	0.9931	0.9938	1.0215	1.0588	1.1912	1.0556
19	0.0526	1.0	0.9934	0.9942	1.0203	1.0556	1.1797	1.0526
20	0.0500	1.0	0.9938	0.9944	1.0193	1.0526	1.1696	1.0500
21	0.0476	1.0	0.9941	0.9947	1.0183	1.0500	1.1605	1.0476
22	0.0455	1.0	0.9943	0.9949	1.0175	1.0476	1.1524	1.0455
23	0.0435	1.0	0.9946	0.9952	1.0167	1.0455	1.1450	1.0435
24	0.0417	1.0	0.9948	0.9954	1.0160	1.0435	1.1383	1.0417
25	0.0400	1.0	0.9950	0.9956	1.0153	1.0417	1.1322	1.0400
26	0.0370	1.0	0.9952	0.9957	1.0147	1.0400	1.1267	1.0385
27	0.0357	1.0	0.9954	0.9959	1.0142	1.0385	1.1215	1.0370
28	0.0345	1.0	0.9955	0.9960	1.0136	1.0370	1.1168	1.0357
29	0.0333	1.0	0.9957	0.9962	1.0132	1.0357	1.1124	1.0345
30	0.0323	1.0	0.9958	0.9963	1.0127	1.0345	1.1084	1.0333
31	0.0313	1.0	0.9960	0.9964	1.0123	1.0333	1.1046	1.0323
32	0.0303	1.0	0.9961	0.9965	1.0119	1.0323	1.1011	1.0312
33	0.0294	1.0	0.9962	0.9966	1.0115	1.0313	1.0978	1.0303
34	0.0286	1.0	0.9963	0.9967	1.0112	1.0303	1.0947	1.0294
35	0.0278	1.0	0.9964	0.9968	1.0109	1.0294	1.0918	1.0286
36	0.0270	1.0	0.9965	0.9969	1.0106	1.0286	1.0891	1.0278
37	0.0263	1.0	0.9966	0.9970	1.0103	1.0278	1.0865	1.0270
38	0.0256	1.0	0.9967	0.9971	1.0100	1.0270	1.0841	1.0263
39	0.0250	1.0	0.9968	0.9972	1.0097	1.0263	1.0818	1.0256
40	0.0244	1.0	0.9969	0.9972	1.0095	1.0256	1.0796	1.0250
41	0.0238	1.0	0.9970	0.9973	1.0093	1.0250	1.0776	1.0244
42	0.0233	1.0	0.9970	0.9974	1.0090	1.0244	1.0756	1.0238
43	0.0227	1.0	0.9971	0.9974	1.0088	1.0238	1.0738	1.0233
44	0.0222	1.0	0.9972	0.9975	1.0086	1.0233	1.0720	1.0227
45	0.0217	1.0	0.9972	0.9975	1.0084	1.0227	1.0703	1.0222

Table 4 Continues								
α	$\beta = 1/\alpha$	μ_{E_t}	Mean of the transformed Distribution (μ_{y_t})					
			SR	WH	ISR	I	IS	S
46	0.0213	1.0	0.9973	0.9976	1.0082	1.0222	1.0687	1.0217
47	0.0208	1.0	0.9973	0.9976	1.0081	1.0217	1.0671	1.0213
48	0.0204	1.0	0.9974	0.9977	1.0079	1.0213	1.0657	1.0208
49	0.0200	1.0	0.9975	0.9977	1.0077	1.0208	1.0643	1.0204
50	0.0196	1.0	0.9975	0.9978	1.0076	1.0204	1.0629	1.0200
51	0.0192	1.0	0.9976	0.9978	1.0074	1.0200	1.0616	1.0196
52	0.0189	1.0	0.9976	0.9979	1.0073	1.0196	1.0604	1.0192
53	0.0185	1.0	0.9976	0.9979	1.0071	1.0192	1.0592	1.0189
54	0.0182	1.0	0.9977	0.9979	1.0070	1.0189	1.0581	1.0185
55	0.0179	1.0	0.9977	0.9980	1.0069	1.0185	1.0570	1.0182
56	0.0175	1.0	0.9978	0.9980	1.0068	1.0182	1.0559	1.0179
57	0.0172	1.0	0.9978	0.9981	1.0066	1.0179	1.0549	1.0175
58	0.0169	1.0	0.9978	0.9981	1.0065	1.0175	1.0539	1.0172
59	0.0167	1.0	0.9979	0.9981	1.0064	1.0172	1.0529	1.0169
60	0.0164	1.0	0.9979	0.9981	1.0063	1.0169	1.0520	1.0167
61	0.0161	1.0	0.9980	0.9982	1.0062	1.0167	1.0511	1.0164
62	0.0159	1.0	0.9980	0.9982	1.0061	1.0164	1.0503	1.0161
63	0.0156	1.0	0.9980	0.9982	1.0060	1.0161	1.0494	1.0159
64	0.0154	1.0	0.9980	0.9983	1.0059	1.0159	1.0486	1.0156
65	0.0152	1.0	0.9981	0.9983	1.0058	1.0156	1.0479	1.0154
66	0.0149	1.0	0.9981	0.9983	1.0057	1.0154	1.0471	1.0152
67	0.0147	1.0	0.9981	0.9983	1.0056	1.0152	1.0464	1.0149
68	0.0145	1.0	0.9982	0.9984	1.0056	1.0149	1.0457	1.0147
69	0.0143	1.0	0.9982	0.9984	1.0055	1.0147	1.0450	1.0145
70	0.0141	1.0	0.9982	0.9984	1.0054	1.0145	1.0443	1.0143
71	0.0139	1.0	0.9982	0.9984	1.0053	1.0143	1.0437	1.0141
72	0.0137	1.0	0.9983	0.9985	1.0052	1.0141	1.0431	1.0139
73	0.0135	1.0	0.9983	0.9985	1.0052	1.0139	1.0424	1.0137
74	0.0133	1.0	0.9983	0.9985	1.0051	1.0137	1.0419	1.0135
75	0.0132	1.0	0.9983	0.9985	1.0050	1.0135	1.0413	1.0133
76	0.0130	1.0	0.9984	0.9985	1.0050	1.0133	1.0407	1.0132
77	0.0128	1.0	0.9984	0.9986	1.0049	1.0132	1.0402	1.0130
78	0.0127	1.0	0.9984	0.9986	1.0048	1.0130	1.0396	1.0128
79	0.0125	1.0	0.9984	0.9986	1.0048	1.0128	1.0391	1.0127
80	0.0123	1.0	0.9984	0.9986	1.0047	1.0127	1.0386	1.0125
81	0.0122	1.0	0.9985	0.9986	1.0047	1.0125	1.0381	1.0123
82	0.0120	1.0	0.9985	0.9986	1.0046	1.0123	1.0377	1.0122
83	0.0119	1.0	0.9985	0.9987	1.0045	1.0122	1.0372	1.0120
84	0.0118	1.0	0.9985	0.9987	1.0045	1.0120	1.0367	1.0119
85	0.0116	1.0	0.9985	0.9987	1.0044	1.0119	1.0363	1.0118
86	0.0115	1.0	0.9985	0.9987	1.0044	1.0118	1.0359	1.0116
87	0.0114	1.0	0.9986	0.9987	1.0043	1.0116	1.0354	1.0115
88	0.0112	1.0	0.9986	0.9987	1.0043	1.0115	1.0350	1.0114

Table 4 Continues								
α	$\beta=1/\alpha$	μ_{E_t}	Mean of the transformed Distribution (μ_{y_t})					
			SR	WH	ISR	I	IS	S
89	0.0111	1.0	0.9986	0.9988	1.0042	1.0114	1.0346	1.0112
90	0.0213	1.0	0.9986	0.9988	1.0042	1.0112	1.0342	1.0111
91	0.0110	1.0	0.9986	0.9988	1.0041	1.0111	1.0338	1.0110
92	0.0109	1.0	0.9986	0.9988	1.0041	1.0110	1.0335	1.0109
93	0.0108	1.0	0.9987	0.9988	1.0041	1.0109	1.0331	1.0108
94	0.0106	1.0	0.9987	0.9988	1.0040	1.0108	1.0327	1.0106
95	0.0105	1.0	0.9987	0.9988	1.0040	1.0106	1.0324	1.0105
96	0.0104	1.0	0.9987	0.9988	1.0039	1.0105	1.0320	1.0104
97	0.0103	1.0	0.9987	0.9989	1.0039	1.0104	1.0317	1.0103
98	0.0102	1.0	0.9987	0.9989	1.0038	1.0103	1.0314	1.0102
99	0.0101	1.0	0.9987	0.9989	1.0038	1.0102	1.0310	1.0101
100	0.0100	1.0	0.9988	0.9989	1.0038	1.0101	1.0307	1.0100

Table 5 : Variances of the untransformed and Transformed Distributions

α	$\beta=1/\alpha$	$\sigma_{E_t}^2$	Variance of the transformed Distribution $(\sigma_{y_t}^2)$					
			SR	WH	ISR	I	IS	S
5	0.2000	0.2000	0.0487	0.0222	0.0755	0.5208	21.7014	1.2480
6	0.1667	0.1667	0.0408	0.0185	0.0584	0.3600	7.5600	0.9722
7	0.1429	0.1429	0.0351	0.0159	0.0476	0.2722	4.0017	0.7930
8	0.1250	0.1250	0.0307	0.0139	0.0401	0.2177	2.5542	0.6680
9	0.1111	0.1111	0.0274	0.0123	0.0346	0.1808	1.8132	0.5761
10	0.1000	0.1000	0.0247	0.0111	0.0304	0.1543	1.3779	0.5060
11	0.0909	0.0909	0.0225	0.0101	0.0271	0.1344	1.0974	0.4508
12	0.0833	0.0833	0.0206	0.0093	0.0245	0.1190	0.9045	0.4062
13	0.0769	0.0769	0.0190	0.0085	0.0223	0.1067	0.7649	0.3696
14	0.0714	0.0714	0.0177	0.0079	0.0205	0.0966	0.6601	0.3389
15	0.0667	0.0667	0.0165	0.0074	0.0190	0.0883	0.5789	0.3129
16	0.0625	0.0625	0.0155	0.0069	0.0176	0.0813	0.5144	0.2905
17	0.0588	0.0588	0.0146	0.0065	0.0165	0.0753	0.4621	0.2711
18	0.0556	0.0556	0.0138	0.0062	0.0155	0.0701	0.4189	0.2541
19	0.0526	0.0526	0.0131	0.0058	0.0146	0.0655	0.3827	0.2391
20	0.0500	0.0500	0.0124	0.0056	0.0138	0.0616	0.3520	0.2258
21	0.0476	0.0476	0.0118	0.0053	0.0130	0.0580	0.3257	0.2138
22	0.0455	0.0455	0.0113	0.0050	0.0124	0.0549	0.3029	0.2030
23	0.0435	0.0435	0.0108	0.0048	0.0118	0.0520	0.2829	0.1933
24	0.0417	0.0417	0.0104	0.0046	0.0113	0.0495	0.2653	0.1845
25	0.0400	0.0400	0.0099	0.0044	0.0108	0.0472	0.2497	0.1764
26	0.0385	0.0385	0.0096	0.0043	0.0103	0.0451	0.2358	0.1690
27	0.0370	0.0370	0.0092	0.0041	0.0099	0.0431	0.2233	0.1622
28	0.0357	0.0357	0.0089	0.0040	0.0096	0.0414	0.2120	0.1559
29	0.0345	0.0345	0.0086	0.0038	0.0092	0.0397	0.2018	0.1501

Table 5 Continues									
α	$\beta=1/\alpha$	$\sigma_{\varepsilon_t}^2$	Variance of the transformed Distribution $\left(\sigma_{y_t}^2\right)$						
			SR	WH	ISR	I	IS	S
30	0.0333	0.0333	0.0083	0.0037	0.0089	0.0382	0.1925	0.1447
31	0.0323	0.0323	0.0080	0.0036	0.0086	0.0368	0.1840	0.1396
32	0.0303	0.0303	0.0078	0.0035	0.0083	0.0355	0.1762	0.1349
33	0.0294	0.0294	0.0075	0.0034	0.0080	0.0343	0.1690	0.1306
34	0.0286	0.0286	0.0073	0.0033	0.0078	0.0332	0.1624	0.1265
35	0.0278	0.0278	0.0071	0.0032	0.0075	0.0321	0.1562	0.1226
36	0.0270	0.0270	0.0069	0.0031	0.0073	0.0311	0.1505	0.1190
37	0.0263	0.0263	0.0067	0.0030	0.0071	0.0302	0.1452	0.1155
38	0.0256	0.0256	0.0066	0.0029	0.0069	0.0293	0.1402	0.1123
39	0.0250	0.0250	0.0064	0.0028	0.0067	0.0285	0.1356	0.1092
40	0.0244	0.0244	0.0062	0.0028	0.0066	0.0277	0.1313	0.1063
41	0.0238	0.0238	0.0061	0.0027	0.0064	0.0269	0.1272	0.1036
42	0.0233	0.0233	0.0059	0.0026	0.0062	0.0262	0.1233	0.1010
43	0.0227	0.0227	0.0058	0.0026	0.0061	0.0256	0.1197	0.0985
44	0.0222	0.0222	0.0057	0.0025	0.0059	0.0249	0.1163	0.0961
45	0.0217	0.0217	0.0055	0.0025	0.0058	0.0243	0.1131	0.0939
46	0.0213	0.0213	0.0054	0.0024	0.0057	0.0237	0.1100	0.0917
47	0.0208	0.0208	0.0053	0.0024	0.0055	0.0232	0.1071	0.0897
48	0.0204	0.0204	0.0052	0.0023	0.0054	0.0227	0.1044	0.0877
49	0.0200	0.0200	0.0051	0.0023	0.0053	0.0222	0.1018	0.0858
50	0.0196	0.0196	0.0050	0.0022	0.0052	0.0217	0.0993	0.0840
51	0.0192	0.0192	0.0049	0.0022	0.0051	0.0212	0.0969	0.0823
52	0.0189	0.0189	0.0048	0.0021	0.0050	0.0208	0.0947	0.0807
53	0.0185	0.0185	0.0047	0.0021	0.0049	0.0204	0.0925	0.0791
54	0.0182	0.0182	0.0046	0.0021	0.0048	0.0200	0.0904	0.0775
55	0.0179	0.0179	0.0045	0.0020	0.0047	0.0196	0.0885	0.0761
56	0.0175	0.0175	0.0045	0.0020	0.0046	0.0192	0.0866	0.0747
57	0.0172	0.0172	0.0044	0.0019	0.0045	0.0188	0.0848	0.0733
58	0.0169	0.0169	0.0043	0.0019	0.0045	0.0185	0.0830	0.0720
59	0.0167	0.0167	0.0042	0.0019	0.0044	0.0182	0.0814	0.0707
60	0.0164	0.0164	0.0042	0.0019	0.0043	0.0178	0.0797	0.0695
61	0.0161	0.0161	0.0041	0.0018	0.0042	0.0175	0.0782	0.0683
62	0.0159	0.0159	0.0040	0.0018	0.0042	0.0172	0.0767	0.0671
63	0.0156	0.0156	0.0040	0.0018	0.0041	0.0169	0.0753	0.0660
64	0.0154	0.0154	0.0039	0.0017	0.0040	0.0166	0.0739	0.0650
65	0.0152	0.0152	0.0038	0.0017	0.0040	0.0164	0.0726	0.0639
66	0.0149	0.0149	0.0038	0.0017	0.0039	0.0161	0.0713	0.0629
67	0.0147	0.0147	0.0037	0.0017	0.0038	0.0159	0.0701	0.0619
68	0.0145	0.0145	0.0037	0.0016	0.0038	0.0156	0.0689	0.0610
69	0.0143	0.0143	0.0036	0.0016	0.0037	0.0154	0.0677	0.0601
70	0.0141	0.0141	0.0036	0.0016	0.0037	0.0151	0.0666	0.0592
71	0.0139	0.0139	0.0035	0.0016	0.0036	0.0149	0.0655	0.0583
72	0.0137	0.0137	0.0035	0.0015	0.0036	0.0147	0.0645	0.0575

Table 5 Continues								
α	$\beta=1/\alpha$	$\sigma_{\varepsilon_t}^2$	Variance of the transformed Distribution $(\sigma_{y_t}^2)$					
			SR	WH	ISR	I	IS	S
73	0.0135	0.0135	0.0034	0.0015	0.0035	0.0145	0.0634	0.0567
74	0.0133	0.0133	0.0034	0.0015	0.0035	0.0143	0.0625	0.0559
75	0.0132	0.0132	0.0033	0.0015	0.0034	0.0141	0.0615	0.0551
76	0.0130	0.0130	0.0033	0.0015	0.0034	0.0139	0.0606	0.0544
77	0.0128	0.0128	0.0032	0.0014	0.0033	0.0137	0.0597	0.0536
78	0.0127	0.0127	0.0032	0.0014	0.0033	0.0135	0.0588	0.0529
79	0.0125	0.0125	0.0032	0.0014	0.0032	0.0133	0.0580	0.0522
80	0.0123	0.0123	0.0031	0.0014	0.0032	0.0131	0.0571	0.0516
81	0.0122	0.0122	0.0031	0.0014	0.0032	0.0130	0.0563	0.0509
82	0.0120	0.0120	0.0030	0.0014	0.0031	0.0128	0.0556	0.0503
83	0.0119	0.0119	0.0030	0.0013	0.0031	0.0126	0.0548	0.0497
84	0.0118	0.0118	0.0030	0.0013	0.0030	0.0125	0.0541	0.0490
85	0.0116	0.0116	0.0029	0.0013	0.0030	0.0123	0.0534	0.0485
86	0.0115	0.0115	0.0029	0.0013	0.0030	0.0122	0.0527	0.0479
87	0.0114	0.0114	0.0029	0.0013	0.0029	0.0120	0.0520	0.0473
88	0.0112	0.0112	0.0028	0.0013	0.0029	0.0119	0.0513	0.0468
89	0.0111	0.0111	0.0028	0.0012	0.0029	0.0118	0.0507	0.0462
90	0.0110	0.0110	0.0028	0.0012	0.0028	0.0116	0.0500	0.0457
91	0.0109	0.0109	0.0027	0.0012	0.0028	0.0115	0.0494	0.0452
92	0.0108	0.0108	0.0027	0.0012	0.0028	0.0114	0.0488	0.0447
93	0.0106	0.0106	0.0027	0.0012	0.0027	0.0112	0.0482	0.0442
94	0.0105	0.0105	0.0027	0.0012	0.0027	0.0111	0.0477	0.0437
95	0.0104	0.0104	0.0026	0.0012	0.0027	0.0110	0.0471	0.0432
96	0.0103	0.0103	0.0026	0.0012	0.0027	0.0109	0.0466	0.0428
97	0.0102	0.0102	0.0026	0.0011	0.0026	0.0107	0.0460	0.0423
98	0.0101	0.0101	0.0025	0.0011	0.0026	0.0106	0.0455	0.0419
99	0.0100	0.0100	0.0025	0.0011	0.0026	0.0105	0.0450	0.0414
100	0.0128	0.0128	0.0025	0.0011	0.0025	0.0104	0.0445	0.0410

Table 6: Relative Change in Variances (RCIV) between the Transformed and the Untransformed Distributions

α	$\beta=1/\alpha$	$\sigma_{\varepsilon_t}^2$	$\text{RCIV} = \dfrac{\sigma_{y_t}^2-\sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$					
			SR	WH	ISR	I	IS	S
9	0.1111	0.1111	-3.0579	-8.0061	-2.2122	0.3855	0.9387	0.8071
10	0.1000	0.1000	-3.0519	-8.0049	-2.2869	0.3520	0.9274	0.8024
11	0.0909	0.0909	-3.0470	-8.0041	-2.3487	0.3238	0.9172	0.7983
12	0.0833	0.0833	-3.0430	-8.0034	-2.4006	0.2998	0.9079	0.7949
13	0.0769	0.0769	-3.0396	-8.0029	-2.4449	0.2790	0.8994	0.7919
14	0.0714	0.0714	-3.0367	-8.0025	-2.4831	0.2609	0.8918	0.7892
15	0.0667	0.0667	-3.0342	-8.0022	-2.5164	0.2450	0.8848	0.7869

Table 6 Continues								
α	$\beta=1/\alpha$	$\sigma_{\varepsilon_t}^2$	$RCIV = \dfrac{\sigma_{y_t}^2 - \sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$					
			SR	WH	ISR	I	IS	S
16	0.0625	0.0625	-3.0320	-8.0019	-2.5457	0.2310	0.8785	0.7849
17	0.0588	0.0588	-3.0301	-8.0017	-2.5716	0.2184	0.8727	0.7830
18	0.0556	0.0556	-3.0284	-8.0015	-2.5947	0.2071	0.8674	0.7814
19	0.0526	0.0526	-3.0268	-8.0014	-2.6155	0.1970	0.8625	0.7799
20	0.0500	0.0500	-3.0255	-8.0012	-2.6342	0.1878	0.8580	0.7785
21	0.0476	0.0476	-3.0242	-8.0011	-2.6512	0.1794	0.8538	0.7773
22	0.0455	0.0455	-3.0231	-8.0010	-2.6667	0.1717	0.8499	0.7761
23	0.0435	0.0435	-3.0221	-8.0009	-2.6809	0.1646	0.8463	0.7751
24	0.0417	0.0417	-3.0212	-8.0008	-2.6939	0.1581	0.8430	0.7741
25	0.0400	0.0400	-3.0203	-8.0008	-2.7059	0.1521	0.8398	0.7732
26	0.0385	0.0385	-3.0195	-8.0007	-2.7170	0.1466	0.8369	0.7724
27	0.0370	0.0370	-3.0188	-8.0007	-2.7273	0.1414	0.8341	0.7716
28	0.0357	0.0357	-3.0181	-8.0006	-2.7368	0.1366	0.8316	0.7709
29	0.0345	0.0345	-3.0175	-8.0006	-2.7458	0.1321	0.8291	0.7702
30	0.0333	0.0333	-3.0169	-8.0005	-2.7541	0.1279	0.8268	0.7696
31	0.0323	0.0323	-3.0163	-8.0005	-2.7619	0.1239	0.8247	0.7690
32	0.0313	0.0313	-3.0158	-8.0005	-2.7692	0.1202	0.8226	0.7684
33	0.0303	0.0303	-3.0153	-8.0004	-2.7761	0.1167	0.8207	0.7679
34	0.0294	0.0294	-3.0149	-8.0004	-2.7826	0.1134	0.8188	0.7674
35	0.0286	0.0286	-3.0144	-8.0004	-2.7887	0.1103	0.8171	0.7669
36	0.0278	0.0278	-3.0140	-8.0004	-2.7945	0.1073	0.8154	0.7665
37	0.0270	0.0270	-3.0137	-8.0004	-2.8000	0.1045	0.8139	0.7661
38	0.0263	0.0263	-3.0133	-8.0003	-2.8052	0.1018	0.8123	0.7657
39	0.0256	0.0256	-3.0129	-8.0003	-2.8101	0.0993	0.8109	0.7653
40	0.0250	0.0250	-3.0126	-8.0003	-2.8148	0.0969	0.8095	0.7649
41	0.0244	0.0244	-3.0123	-8.0003	-2.8193	0.0946	0.8082	0.7646
42	0.0238	0.0238	-3.0120	-8.0003	-2.8235	0.0924	0.8070	0.7642
43	0.0233	0.0233	-3.0117	-8.0003	-2.8276	0.0903	0.8058	0.7639
44	0.0227	0.0227	-3.0115	-8.0003	-2.8315	0.0883	0.8046	0.7636
45	0.0222	0.0222	-3.0112	-8.0002	-2.8352	0.0864	0.8035	0.7633
46	0.0217	0.0217	-3.0110	-8.0002	-2.8387	0.0846	0.8024	0.7630
47	0.0213	0.0213	-3.0107	-8.0002	-2.8421	0.0829	0.8014	0.7628
48	0.0208	0.0208	-3.0105	-8.0002	-2.8454	0.0812	0.8004	0.7625
49	0.0204	0.0204	-3.0103	-8.0002	-2.8485	0.0796	0.7995	0.7623
50	0.0200	0.0200	-3.0101	-8.0002	-2.8515	0.0780	0.7986	0.7620
51	0.0196	0.0196	-3.0099	-8.0002	-2.8544	0.0765	0.7977	0.7618
52	0.0192	0.0192	-3.0097	-8.0002	-2.8571	0.0751	0.7968	0.7616
53	0.0189	0.0189	-3.0095	-8.0002	-2.8598	0.0737	0.7960	0.7614
54	0.0185	0.0185	-3.0093	-8.0002	-2.8624	0.0724	0.7952	0.7612
55	0.0182	0.0182	-3.0092	-8.0002	-2.8649	0.0711	0.7945	0.7610
56	0.0179	0.0179	-3.0090	-8.0002	-2.8672	0.0698	0.7937	0.7608
57	0.0175	0.0175	-3.0088	-8.0001	-2.8696	0.0686	0.7930	0.7606

Table 6 Continues

α	$\beta=1/\alpha$	$\sigma_{\varepsilon_t}^2$	$\mathrm{RCIV} = \frac{\sigma_{y_t}^2-\sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$					
			SR	WH	ISR	I	IS	S
58	0.0172	0.0172	-3.0087	-8.0001	-2.8718	0.0675	0.7923	0.7604
59	0.0169	0.0169	-3.0085	-8.0001	-2.8739	0.0664	0.7917	0.7603
60	0.0167	0.0167	-3.0084	-8.0001	-2.8760	0.0653	0.7910	0.7601
61	0.0164	0.0164	-3.0082	-8.0001	-2.8780	0.0642	0.7904	0.7599
62	0.0161	0.0161	-3.0081	-8.0001	-2.8800	0.0632	0.7898	0.7598
63	0.0159	0.0159	-3.0080	-8.0001	-2.8819	0.0622	0.7892	0.7596
64	0.0156	0.0156	-3.0079	-8.0001	-2.8837	0.0613	0.7886	0.7595
65	0.0154	0.0154	-3.0077	-8.0001	-2.8855	0.0604	0.7880	0.7593
66	0.0152	0.0152	-3.0076	-8.0001	-2.8872	0.0595	0.7875	0.7592
67	0.0149	0.0149	-3.0075	-8.0001	-2.8889	0.0586	0.7870	0.7591
68	0.0147	0.0147	-3.0074	-8.0001	-2.8905	0.0577	0.7865	0.7589
69	0.0145	0.0145	-3.0073	-8.0001	-2.8921	0.0569	0.7860	0.7588
70	0.0143	0.0143	-3.0072	-8.0001	-2.8936	0.0561	0.7855	0.7587
71	0.0141	0.0141	-3.0071	-8.0001	-2.8951	0.0554	0.7850	0.7586
72	0.0139	0.0139	-3.0070	-8.0001	-2.8965	0.0546	0.7845	0.7585
73	0.0137	0.0137	-3.0069	-8.0001	-2.8980	0.0539	0.7841	0.7583
74	0.0135	0.0135	-3.0068	-8.0001	-2.8993	0.0531	0.7837	0.7582
75	0.0133	0.0133	-3.0067	-8.0001	-2.9007	0.0524	0.7832	0.7581
76	0.0132	0.0132	-3.0066	-8.0001	-2.9020	0.0518	0.7828	0.7580
77	0.0130	0.0130	-3.0065	-8.0001	-2.9032	0.0511	0.7824	0.7579
78	0.0128	0.0128	-3.0064	-8.0001	-2.9045	0.0505	0.7820	0.7578
79	0.0127	0.0127	-3.0064	-8.0001	-2.9057	0.0498	0.7816	0.7577
80	0.0125	0.0125	-3.0063	-8.0001	-2.9068	0.0492	0.7813	0.7576
81	0.0123	0.0123	-3.0062	-8.0001	-2.9080	0.0486	0.7809	0.7575
82	0.0122	0.0122	-3.0061	-8.0001	-2.9091	0.0480	0.7805	0.7574
83	0.0120	0.0120	-3.0061	-8.0001	-2.9102	0.0475	0.7802	0.7574
84	0.0119	0.0119	-3.0060	-8.0001	-2.9112	0.0469	0.7798	0.7573
85	0.0118	0.0118	-3.0059	-8.0001	-2.9123	0.0464	0.7795	0.7572
86	0.0116	0.0116	-3.0058	-8.0001	-2.9133	0.0458	0.7792	0.7571
87	0.0115	0.0115	-3.0058	-8.0001	-2.9143	0.0453	0.7789	0.7570
88	0.0114	0.0114	-3.0057	-8.0001	-2.9152	0.0448	0.7785	0.7570
89	0.0112	0.0112	-3.0056	-8.0001	-2.9162	0.0443	0.7782	0.7569
90	0.0111	0.0111	-3.0056	-8.0001	-2.9171	0.0438	0.7779	0.7568
91	0.0110	0.0110	-3.0055	-8.0001	-2.9180	0.0434	0.7776	0.7567
92	0.0109	0.0109	-3.0055	-8.0001	-2.9189	0.0429	0.7774	0.7567
93	0.0108	0.0108	-3.0054	-8.0001	-2.9198	0.0424	0.7771	0.7566
94	0.0106	0.0106	-3.0053	-8.0001	-2.9206	0.0420	0.7768	0.7565
95	0.0105	0.0105	-3.0053	-8.0001	-2.9215	0.0416	0.7765	0.7564
96	0.0104	0.0104	-3.0052	-8.0001	-2.9223	0.0411	0.7763	0.7564
97	0.0103	0.0103	-3.0052	-8.0001	-2.9231	0.0407	0.7760	0.7563
98	0.0102	0.0102	-3.0051	-8.0001	-2.9239	0.0403	0.7757	0.7563
99	0.0101	0.0101	-3.0051	-8.0000	-2.9246	0.0399	0.7755	0.7562
100	0.0100	0.0100	-3.0050	-8.0000	-2.9254	0.0395	0.7752	0.7561

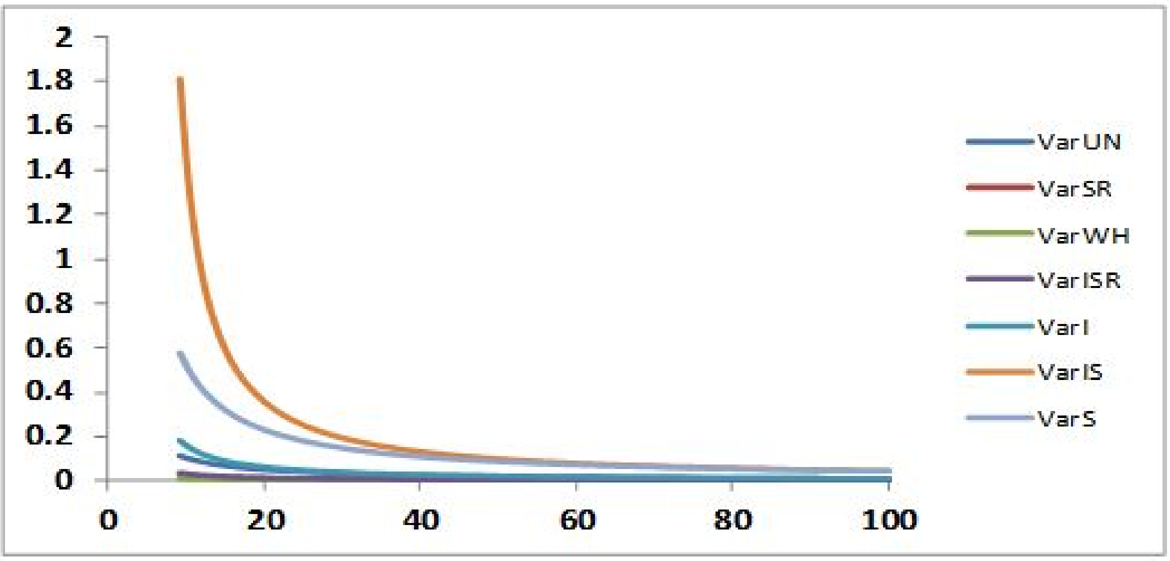

Figure 1: Plots of the Untransformed and the transformed Variances

Legend:
Var UN = Variance of the untransformed distribution
VarSR = Variance of the Square Root transformed distribution
VarWH = Variance of the Wilson Hilferty transformed distribution
VarISR = Variance of the Inverse Square Root transformed distribution
VarI = Variance of the Inverse transformed distribution
VarIS = Variance of the Inverse Square transformed distribution
VarS = Variance of the square transformed distribution

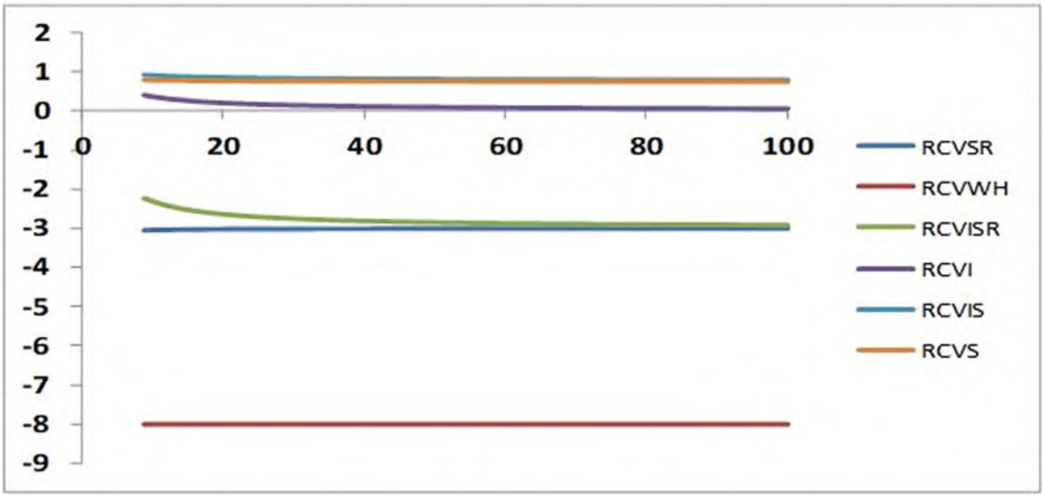

Figure 2: Plots of the Relative Change in Variance (RCIV) against the Shape Parameter(α)

Legend:
RCVSR = RCIV between the Variance of the untransformed and square root transformed distributions.
RCVWH = RCIV between the Variance of the untransformed and Wilson Hilferty transformed distributions.
RCVISR = RCIV between the Variance of the untransformed and inverse square root transformed distributions
RCVI = RCIV between the Variance of the untransformed and inverse transformed distributions.
RCVIS = RCIV between the Variance of the untransformed and Inverse square transformed distributions.
RCVS = RCIV between the Variance of the untransformed and Square transformed distributions.